A small air turbine with an isentropic efficiency of 80\% should produce 270 kJ/kg of work. The inlet temperature is 1000 K, and the turbine exhausts to the atmosphere. Find the required inlet pressure and exhaust temperature.

**Given:** Inlet temperature, outlet pressure, specific work, isentropic efficiency  
**Find:** Inlet pressure, Exit temperature  
**Assumptions:** Ideal Gas. Constant Specific Heat. Ambient (exit) pressure is 100 kPa. Adiabatic  
**Solution:**  
First Law:  
\[
0 = \dot{Q} - \dot{W} + \dot{m} (h_i - h_e)
\]
\[
\frac{\dot{W}}{\dot{m}} = w = h_i - h_e
\]
\[
w = C_p (T_i - T_e)
\]
\[
w_a = C_p (T_i - T_{ea})
\]
\[
T_{ea} = T_i - \frac{w_a}{C_p}
\]
\[
T_{ea} = 1000 \text{ K} - \frac{270 \text{ kJ/kg}}{1.004 \text{ kJ/kgK}}
\]
\[
T_{ea} = 728.9 \text{ K}
\]

Need to look at the ideal case to determine inlet pressure.  
\[
w_s = \frac{w_a}{\eta_T} = C_p (T_i - T_{ea})
\]
\[
T_{es} = T_i - \frac{w_s}{C_p}
\]
\[
T_{es} = 1000 \text{ K} - \frac{270 \text{ kJ/kg}}{1.004 \text{ kJ/kgK}}/0.8
\]
\[
T_{es} = 661 \text{ K}
\]

Ideal case is isentropic  
\[
\frac{P_i}{P_e} = \left( \frac{T_i}{T_{es}} \right)^{k/k-1}
\]
\[
P_i = P_e \left( \frac{T_i}{T_{es}} \right)^{k/k-1}
\]
\[
P_i = 100 \text{ kPa} \left( \frac{1000 \text{ K}}{661 \text{ K}} \right)^{1.4/0.4}
\]
\[
P_i = 426
\]

We could also use the reversible work equation to find $T_{es}$:  
\[
w_s = -\frac{kR}{k-1} (T_{es} - T_i)
\]
\[
T_{es} = -w_s k - \frac{1}{kR} + T_i
\]
To approximate an actual spark-ignition engine, consider an air-standard Otto cycle that has a heat addition of 1800 kJ/kg of air, a compression ratio of 7, and a pressure and temperature at the beginning of the compression process of 90 kPa and 10°C. Assuming constant specific heat, determine the maximum pressure and temperature of the cycle, the thermal efficiency of the cycle, and the mean effective pressure.

**Given:** Otto cycle. Pre-compression pressure and temperature. Specific heat addition.

**Find:** Maximum Temperature and Pressure. Thermal Efficiency. MEP

**Solution:**

Setting up states.

<table>
<thead>
<tr>
<th>State 1</th>
<th>State 2</th>
<th>State 3</th>
<th>State 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_1 = 283$ K</td>
<td>$v_2 = v_1/7 = 0.129 \text{ m}^3/\text{kg}$</td>
<td>$v_3 = v_1/7 = 0.129 \text{ m}^3/\text{kg}$</td>
<td>$v_4 = v_1 = 0.9025 \text{ m}^3/\text{kg}$</td>
</tr>
<tr>
<td>$P_1 = 90$ kPa</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$v_1 = RT_1/P_1 = 0.9025 \text{ m}^3/\text{kg}$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Process $1 \rightarrow 2$ is isentropic.

$$T_2 = T_1 \left( \frac{v_1}{v_2} \right)^{k-1}$$

$$T_2 = 283 \text{ K (7)}^{0.4}$$

$$T_2 = 616.3 \text{ K}$$

Process $2 \rightarrow 3$ is constant volume.

First Law:

$$u_3 - u_2 = q_{23} - w_{23}$$

$$C_v (T_3 - T_2) = q_{23} q_{23}$$

$$T_3 = T_2 + \frac{q_{in}}{C_v}$$

$$T_3 = 616.3 \text{ K} + \frac{1800 \text{ kJ/kg}}{0.717 \text{ kJ/kgK}}$$

$$T_3 = 3127 \text{ K} \leftarrow T_{\text{max}}$$

$$P_3 = RT_3/v_3 = 6957 \text{ kPa} \leftarrow P_{\text{max}}$$

Process $3 \rightarrow 4$ is isentropic.

$$T_4 = T_3 \left( \frac{v_3}{v_4} \right)^{k-1}$$

$$T_4 = 3127 \text{ K (1/7)}^{0.4}$$

$$T_4 = 1436 \text{ K}$$
All states are now defined. Find efficiency.

\[ \eta_{th} = \frac{w_{net}}{q_{in}} = 1 - \frac{q_{out}}{q_{in}} \]

\[ q_{out} = -q_{41} \]

Apply first law for \( 4 \rightarrow 1 \):

\[ u_{1} - u_{4} = q_{41} - w_{41} \]

\[ q_{out} = -q_{41} = u_{4} - u_{1} = C_{v}(T_{4} - T_{1}) = 0.717 \frac{kJ}{kgK} (1436 K - 283 K) = 826.7 \frac{kJ}{kg} \]

\[ \eta_{th} = 1 - \frac{q_{out}}{q_{in}} \]

\[ \eta_{th} = 1 - \frac{826.7 \frac{kJ}{kg}}{1800 \frac{kJ}{kg}} \]

\[ \eta_{th} = 54.1\% \]

Find MEP.

\[ MEP = \frac{w_{net}}{v_{max} - v_{min}} = \frac{q_{in} - q_{out}}{v_{1} - v_{2}} = \frac{1800 \frac{kJ}{kg} - 826.7 \frac{kJ}{kg}}{0.9025 \frac{m^3}{kg} - 0.129 \frac{m^3}{kg}} = 1258 \text{ kPa} \]
A diesel engine has a state before compression of 95 kPa, 290 K, a peak pressure of 6000 kPa, and a maximum temperature of 2400K. Find the volumetric compression ratio and the thermal efficiency.

**Given:** Pre-compression state. Post combustion state.

**Find:** Compression ratio. Efficiency

**Solution:**
Setting up states. Process 1 → 2 is isentropic so,

<table>
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<tr>
<th>State 1</th>
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<th>State 3</th>
<th>State 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>T_1 = 290 K</td>
<td>T_2 = 2400 K</td>
<td>T_3 = 2400 K</td>
<td>T_4 = 2400 K</td>
</tr>
<tr>
<td>P_1 = 95 kPa</td>
<td>P_2 = 6000 kPa</td>
<td>P_3 = 6000 kPa</td>
<td>P_4 = 95 kPa</td>
</tr>
<tr>
<td>v_1 = RT_1/P_1 = 0.8761 m^3/kg</td>
<td>v_2 = RT_2/P_2 = 0.453 m^3/kg</td>
<td>v_3 = RT_3/P_3 = 0.1148 m^3/kg</td>
<td>v_4 = v_1 = 0.8761 m^3/kg</td>
</tr>
</tbody>
</table>

\[
T_2 = T_1 \left( \frac{P_2}{P_1} \right)^{k-1/k}
\]

\[
T_2 = 290 K \left( \frac{6000 \text{ kPa}}{95 \text{ kPa}} \right)^{0.4/1.4}
\]

\[
T_2 = 948 K v_2 = R * T_2/P_2 = \left( \frac{0.287 \text{ kJ/kgK}}{6000 \text{ kPa}} \right) (948 \text{ K})
\]

\[
v_2 = 0.0453 \text{ m}^3/\text{kg}
\]

Compression Ratio = \[
\frac{v_1}{v_2} = \frac{0.8761 \text{ m}^3/\text{kg}}{0.0453 \text{ m}^3/\text{kg}} = 19.3
\]

Process 3 → 4 is isentropic so,

\[
\frac{T_4}{T_3} = \left( \frac{v_3}{v_4} \right)^{k-1}
\]

\[
T_4 = T_3 \left( \frac{v_3}{v_4} \right)^{k-1}
\]

\[
T_4 = 2400 K \left( \frac{0.1148 \text{ m}^3/\text{kg}}{0.8761} \right)^{0.4} = 1064.5 \text{ K}
\]

Find work and heat for each process.

1 → 2 First Law:

\[
u_2 - u_1 = \varphi \epsilon - w_{12}
\]

\[
w_{12} = u_1 - u_2 = C_v (T_1 - T_2) = 0.717 \frac{\text{kJ}}{\text{kgK}} (290 \text{ K} - 948 \text{ K})
\]

\[
w_{12} = -471.8 \frac{\text{kJ}}{\text{kg}}
\]
2 → 3 First Law:

\[ u_3 - u_2 = q_{23} - w_{23} \]

\[ w_{23} = P(\nu_3 - \nu_2) = 6000 \text{ kPa} \left(0.1148 \frac{m^3}{kg} - 0.0453 \frac{m^3}{kg}\right) w_{23} = 416.7 \frac{kJ}{kg} q_{23} = u_3 - u_2 + w_{23} \]

\[ q_{23} = u_3 - u_2 + P(\nu_3 - \nu_2) \]

\[ q_{23} = u_3 - u_2 + P_3 \nu_3 - P_2 \nu_2 \]

\[ q_{23} = (u_3 + P_3 \nu_3) - (u_2 + P_2 \nu_2) \]

\[ q_{23} = h_3 - h_2 = C_p(T_3 - T_2) = 1.004 \frac{kJ}{kgK} (2400 \text{ K} - 948 \text{ K}) q_{23} = 1458 \frac{kJ}{kgK} \]

3 → 4 First Law:

\[ u_4 - u_3 = q_{34} - w_{34} \]

\[ w_{34} = u_3 - u_4 = C_v (T_3 - T_4) = 0.717 \frac{kJ}{kgK} (2400 \text{ K} - 1064.5 \text{ K}) \]

\[ w_{34} = 958 \frac{kJ}{kgK} \]

4 → 1 First Law:

\[ u_1 - u_4 = q_{41} - w_{41} \]

\[ q_{41} = u_1 - u_4 = C_v (T_1 - T_4) = 0.717 \frac{kJ}{kgK} (290 \text{ K} - 1064.5 \text{ K}) \]

\[ q_{41} = -555.4 \frac{kJ}{kgK} \]

Now find thermal efficiency.

\[ \eta_h = \frac{w_{\text{net}}}{q_{\text{in}}} = w_{12} + w_{23} + w_{34} + w_{41} = -471.8 \frac{kJ}{kg} + 416.8 \frac{kJ}{kg} + 957.5 \frac{kJ}{kg} + 0 \]

\[ w_{\text{net}} = 902.5 \frac{kJ}{kg} \]

\[ q_{\text{in}} = q_{23} = 1547.8 \frac{kJ}{kg} \]

\[ \eta_h = \frac{902.5 \frac{kJ}{kg}}{1547.8 \frac{kJ}{kg}} = 61.9\% \]
Consider a solar-energy-powered ideal Rankine cycle that uses water as the working fluid. Saturated vapor leaves the solar collector at 175°C, and the condenser pressure is 10 kPa. Determine the thermal efficiency of this cycle.

**Given:** Ideal Rankine Cycle. State entering turbine, condenser pressure

**Find:** Thermal Efficiency of the cycle

**Assumptions:** Steady flow

**Solution:**

Define states: Now calculate efficiency:

<table>
<thead>
<tr>
<th>State 1</th>
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<th>State 3</th>
<th>State 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1 = 0$</td>
<td>$P_2 = P_3 = 892$ kPa</td>
<td>$T_3 = 175^\circ$C</td>
<td>$P_4 = 10$ kPa</td>
</tr>
<tr>
<td>$P_1 = 10$ kPa</td>
<td>$h_2 = h_1 + v_1 (P_2 - P_1)$</td>
<td>$x_3 = 1$</td>
<td>$s_4 = s_3$</td>
</tr>
<tr>
<td>Below are from CATT3</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$v_1 = 0.00101\text{m}^3\text{kg}^{-1}$  
$h_1 = 191.8\text{kJkg}^{-1}$  
$h_1 = 0.00101\text{m}^3\text{kg}^{-1}$ (892 kPa - 10 kPa)

$h_2 = 192.7\text{kJkg}^{-1}$  
$h_2 = 191.8\text{kJkg}^{-1} + 0.00101\text{m}^3\text{kg}^{-1}$

$h_3 = 892$ kPa  
$h_3 = 2774\text{kJkg}^{-1}$

$s_3 = 6.626\text{kJkg}^{-1}$  
$s_3 = 6.626\text{kJkg}^{-1}$

$s_4 = 2098\text{kJkg}^{-1}$  
$s_4 = 2098\text{kJkg}^{-1}$

$\eta = w_{\text{net}} = \frac{w_{12} + w_{34}}{q_{23}} = \frac{(h_1 - h_2) + (h_3 - h_4)}{h_3 - h_2}$

$= \frac{(191.8\text{kJkg}^{-1} - 192.7\text{kJkg}^{-1}) + 2774\text{kJkg}^{-1} - 2098\text{kJkg}^{-1}}{2774\text{kJkg}^{-1} - 192.7\text{kJkg}^{-1}}$

$\eta = 26\%$
A steam power cycle (Rankine Cycle) has a high pressure of 3 MPa and a condenser exit temperature of 45°C. The turbine efficiency is 85%, and other cycle components are ideal. If the boiler superheats to 800°C, find the cycle efficiency.

**Given:** Boiler Pressure, Boiler outlet temperature, Condenser exit temperature, Turbine efficiency.

**Find:** Cycle efficiency

**Assumptions:** Steady Flow

**Define states:**

<table>
<thead>
<tr>
<th>State 1</th>
<th>State 2</th>
<th>State 3</th>
<th>State 4s</th>
<th>State 4a</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1 = 0$</td>
<td>$P_2 = 3000$ kPa</td>
<td>$h_2 = h_1 + v_1 (P_2 - P_1)$</td>
<td>$P_3 = 3000$ kPa</td>
<td>$P_4 = P_1 = 9.59$ kPa</td>
</tr>
<tr>
<td>$P_1 = P_{Sat}</td>
<td>_{T=45°C} = 9.59$ kPa</td>
<td>$T_3 = 800$°C</td>
<td>$s_{4s} = s_3$</td>
<td></td>
</tr>
<tr>
<td>$P_4 = P_1 = 9.59$ kPa</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Below are from CATT3

- $v_1 = 0.00101 \text{ m}^3/\text{kg}$
- $h_1 = 188.4 \text{ kJ/kg}$

Use turbine efficiency to find State 4a.

$$\eta_T = \frac{w_a}{w_s} = \frac{h_3 - h_{4s}}{h_3 - h_{4a}}$$

$$w_a = \eta_T w_s = \eta_T (h_3 - h_{4s}) = 0.85 \left( \frac{4146 \text{ kJ/kg}}{2526 \text{ kJ/kg}} \right) = 0.85 \left( \frac{1620 \text{ kJ/kg}}{kg} \right) = 1377 \text{ kJ/kg}$$

$$h_{4a} = h_3 - w_a = 4146 \text{ kJ/kg} - 1377 \text{ kJ/kg} = 2769 \text{ kJ/kg}$$

Now find cycle efficiency.

$$\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{w_{12} + w_{34a}}{q_{23}} = \frac{(h_1 - h_2) + (h_3 - h_{4a})}{h_3 - h_2} = \frac{(188.4 \text{ kJ/kg} - 191.4 \text{ kJ/kg}) + (4146 \text{ kJ/kg} - 2769 \text{ kJ/kg})}{4146 \text{ kJ/kg} - 191.4 \text{ kJ/kg}} = \frac{1374 \text{ kJ/kg}}{3955 \text{ kJ/kg}}$$

$$\eta_{th} = 34.7\%$$