Two rigid tanks each contain 10 kg of N\(_2\) gas at 1000 K, 500 kPa. They are now thermally connected to a reversible heat pump, which heats one and cools the other with no heat transfer to the surroundings. When one tank is heated to 1500 K, the process stops. Find the final (P,T) of both tanks and the work input to the heat pump, assuming constant heat capacities.

**Given:** \(P_{A,1}, P_{B,1}, T_{A,1}, T_{B,1}, T_{B,2}\)

**Find:** \(P_{A,2}, P_{B,2}, T_{A,2}, T_{B,2}\)

**Assumptions:** Nitrogen can be treated as an ideal-gas. System is closed.

**Solution:**

Second Law:

\[
S_2 - S_1 = \int_1^2 \frac{\delta Q}{T} + S_i - S_e + S_{gen}
\]

\[
S_2 - S_1 = \int_1^2 \frac{\delta Q}{T} + S_i - S_e + S_{gen}
\]

\[
m_A(s_2 - s_1)_A + m_B(s_2 - s_1)_B = 0
\]

\[
0 = m_A \left[ C_v \ln \left( \frac{T_{A,2}}{T_{A,1}} \right) + R \ln \left( \frac{v_{A,2}}{v_{A,1}} \right) \right] + m_B \left[ C_v \ln \left( \frac{T_{B,2}}{T_{B,1}} \right) + R \ln \left( \frac{v_{B,2}}{v_{B,1}} \right) \right]
\]

\[
0 = m_A \left[ C_v \ln \left( \frac{T_{A,2}}{T_{A,1}} \right) + R \ln \left( \frac{v_{A,2}}{v_{A,1}} \right) \right] + m_B \left[ C_v \ln \left( \frac{T_{B,2}}{T_{B,1}} \right) + R \ln \left( \frac{v_{B,2}}{v_{B,1}} \right) \right]
\]

\[
m_A = m_B
\]

\[
0 = C_v \ln \left( \frac{T_{A,2}}{T_{A,1}} \right) + C_v \ln \left( \frac{T_{B,2}}{T_{B,1}} \right)
\]

\[
0 = \ln \left( \frac{T_{A,2}}{T_{A,1}} \right) + \ln \left( \frac{T_{B,2}}{T_{B,1}} \right)
\]

\[
\ln \left( \frac{T_{A,2}}{T_{A,1}} \right) = - \ln \left( \frac{T_{B,2}}{T_{B,1}} \right)
\]

\[
T_{A,2} = T_{A,1} \left( \frac{T_{B,1}}{T_{B,2}} \right) = 1000 \text{ K} \frac{1000 \text{ K}}{1500 \text{ K}}
\]

\[
T_{A,2} = 667 \text{ K}
\]

Now find pressures.

\[
R = \frac{P v}{T} = \frac{P_1 v_1}{T_1} = \frac{P_2 v_2}{T_2}
\]

Volume is fixed.

\[
P_2 = P_1 \frac{T_2}{T_1}
\]

\[
P_{2,B} = 500 \text{ kPa} \left( \frac{1500 \text{ K}}{1000 \text{ K}} \right) = 750 \text{ kPa}
\]

\[
P_{2,A} = 500 \text{ kPa} \left( \frac{667 \text{ K}}{1000 \text{ K}} \right) = 333 \text{ kPa}
\]
Apply first law to find work

\[
U_2 - U_1 = Q_{12} - W_{12} + \Theta_i - \Theta_e \\
W_{12} = U_1 - U_2 \\
W_{12} = m_A (u_1 - u_2)_A + m_B (u_1 - u_2)_B \\
W_{12} = m_A C_v (T_1 - T_2)_A + m_B C_v (T_1 - T_2)_B \\
W_{12} = (10 \text{ kg}) 0.745 \frac{\text{kJ}}{\text{kgK}} (667 \text{ K} - 1000 \text{ K}) + (10 \text{ kg}) 0.745 \frac{\text{kJ}}{\text{kgK}} (1500 \text{ K} - 1000 \text{ K})
\]

\[
W_{12} = 1244 \text{ kJ}
\]
A cylinder fitted with a movable piston contains water at 3 MPa with 50% quality, at which point the volume is 20 L. The water now expands to 1.2 MPa as a result of receiving 600 kJ of heat from a large source at 300°C. It is claimed that the water does 124 kJ of work during this process. Is this possible?

**Given:** Initial state, heat, work, and final pressure for a process on a closed system.

**Find:** Determine if the process is possible.

**Assumptions:** Closed System

**Solution:**

Define States:

<table>
<thead>
<tr>
<th>State 1</th>
<th>State 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_1 = 3000 \text{ kPa} )</td>
<td>( P_2 = 1200 \text{ kPa} )</td>
</tr>
<tr>
<td>( x_1 = 0.5 )</td>
<td></td>
</tr>
<tr>
<td>( V_1 = 0.02 \text{ m}^3 )</td>
<td></td>
</tr>
<tr>
<td>From CATT3</td>
<td></td>
</tr>
<tr>
<td>( v_1 = 0.03395 \text{ m}^3/\text{kg} )</td>
<td></td>
</tr>
<tr>
<td>( u_1 = 1804 \text{ kJ/kg} )</td>
<td></td>
</tr>
<tr>
<td>( s_1 = 4.416 \text{ kJ/kgK} )</td>
<td></td>
</tr>
<tr>
<td>( m = V_1/v_1 = 0.589 \text{ kg} )</td>
<td></td>
</tr>
</tbody>
</table>

Apply First Law:

\[
m(u_2 - u_1) = Q_{12} - W_{12}
\]

\[
u_2 = \frac{(Q_{12} - W_{12})}{m} + u_1
\]

\[
u_2 = \frac{(600 \text{ kJ} - 124 \text{ kJ})}{0.589 \text{ kg}} + 1804 \text{ kJ/kg}
\]

\[
u_2 = 2612 \text{ kJ/kg}
\]

Use \( P_2 \) and \( u_2 \) to find \( s_2 \) and then apply second law:

\[
s_2 = 6.588 \text{ kJ/kgK} \quad \text{(Found by iterating with CATT3)}
\]

\[
S_2 - S_1 = \int_1^2 \frac{\delta Q}{T} + S' - S' + S_{gen}
\]

\[
m \left( s_2 - s_1 \right) = -\frac{Q_{12}}{T_{source}} + S_{gen}
\]

\[
S_{gen} = m \left( s_2 - s_1 \right) - \frac{Q_{12}}{T_{source}}
\]

\[
S_{gen} = 0.589 \text{ kg} \left( 6.588 \text{ kJ/kgK} - 4.416 \text{ kJ/kgK} \right) - \frac{600 \text{ kJ}}{573 \text{ K}}
\]

\[
S_{gen} = 0.232 \text{ kJ/K}
\]

\( S_{gen} > 0 \) so process is possible.
One kilogram of air at 300 K is mixed with 1 kg of air at 400 K in a process at a constant 100 kPa and 
\( Q = 0 \). Find the final \( T \) and the entropy generation in the process.

**Given:** Initial temperature of air, constant pressure process, reaches uniform temperature.  
**Find:** \( T_{\text{final}} \) and \( S_{\text{gen}} \)  
**Assumptions:** Ideal gas, constant specific heat.  
**Solution:**

Apply First Law:

\[
U_2 - U_1 = Q_{12} - W_{12} \\
U_2 - U_1 = -\int_1^2 P \, dV \\
U_2 - U_1 = -P(V_2 - V_1) \\
(U_2 + PV_2) - (U_1 + PV_1) = 0 \\
H_2 - H_1 = 0 \\
\frac{(H_2 - H_1)_A + (H_2 - H_1)_B}{2} = 0 \\
m_A C_p (T_{A,2} - T_{A,1}) - m_B C_p (T_{B,2} - T_{B,1}) = 0 \\
T_2 = \frac{T_{A,2} + T_{B,2}}{2} \\
m_A C_p (T_2 - T_{A,1}) - m_B C_p (T_2 - T_{B,1}) = 0 \\
T_2 = \frac{m_A C_p T_{A,1} + m_B C_p T_{B,1}}{m_A + m_B} \\
m_A = m_B \\
T_2 = \frac{T_{A,1} + T_{B,1}}{2} = 350 \text{ K} \\
\]

Apply Second Law:

\[
S_2 - S_1 = \int_1^2 \frac{\delta Q}{T} + S_{\text{gen}} \\
m_A (s_2 - s_1) + m_A (s_2 - s_1) = S_{\text{gen}} \\
S_{\text{gen}} = m_A \left( C_p \ln \left( \frac{T_2}{T_{A,1}} \right) - R \ln \left( \frac{P_2}{P_1} \right) \right) + m_B \left( C_p \ln \left( \frac{T_2}{T_{B,1}} \right) - R \ln \left( \frac{P_2}{P_1} \right) \right) \\
S_{\text{gen}} = 1 \text{ kg} \left( \frac{1.004 \text{ kJ}}{\text{kgK}} \ln \left( \frac{350 \text{ K}}{300 \text{ K}} \right) \right) + 1 \text{ kg} \left( \frac{1.004 \text{ kJ}}{\text{kgK}} \ln \left( \frac{350 \text{ K}}{400 \text{ K}} \right) \right) \\
S_{\text{gen}} = 1 \text{ kg} \left( \frac{1.004 \text{ kJ}}{\text{kgK}} \left[ \ln \left( \frac{350 \text{ K}}{300 \text{ K}} \right) + \ln \left( \frac{350 \text{ K}}{400 \text{ K}} \right) \right] \right) \\
S_{\text{gen}} = 0.0207 \frac{\text{kJ}}{\text{K}} \\
\]
Steam enters a turbine at 3 MPa, 450°C, expands in a reversible adiabatic processes, and exhausts at 10 kPa. Changes in kinetic and potential energies between the inlet and the exit of the turbine are small. The power output of the turbine is 800 kW. What is the mass flow rate of the steam through the turbine?

**Given:** Inlet pressure and temperature, outlet pressure, adiabatic, reversible, power output.

**Find:** The mass flow rate through the turbine.

**Assumptions:** Steady Flow.

**Solution:**

Apply First Law,

\[ 0 = \dot{Q} - \dot{W} + \dot{m} (h_i - h_e) \]

\[ \dot{m} = \frac{\dot{W}}{h_i - h_e} \]

Apply Second Law,

\[ 0 = \dot{S}_{gen} + \dot{m} (s_i - s_e) \]

\[ s_e = s_i \]

Define States:

<table>
<thead>
<tr>
<th>State i</th>
<th>State e</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_i = 3000$ kPa</td>
<td>$P_e = 10$ kPa</td>
</tr>
<tr>
<td>$T_i = 450$°C</td>
<td>$s_e = s_i = 7.083 \frac{kJ}{kgK}$</td>
</tr>
<tr>
<td>From CATT3</td>
<td></td>
</tr>
<tr>
<td>$s_i = 7.083 \frac{kJ}{kgK}$</td>
<td></td>
</tr>
<tr>
<td>$h_i = 3344 \frac{kJ}{kg}$</td>
<td>$h_e = 2244 \frac{kJ}{kg}$</td>
</tr>
</tbody>
</table>

\[ \dot{m} = \frac{\dot{W}}{h_i - h_e} \]

\[ \dot{m} = \frac{800 \text{ kW}}{3344 \frac{kJ}{kg} - 2244 \frac{kJ}{kg}} = 0.727 \frac{kg}{s} \]
A diffuser is a steady-state device in which a fluid flowing at high velocity is decelerated such that the pressure increases in the process. Air at 120 kPa, 30°C enters diffuser with a velocity of 200 m/s and exits with a velocity of 20 m/s. Assuming the process is reversible and adiabatic, what are the exit pressure and temperature of the air?

**Given:** Inlet pressure, temperature and velocity. Exit velocity. Reversible. Adiabatic.

**Find:** Exit pressure and temperature.

**Assumptions:** Ideal Gas with constant specific heat.

Apply First Law,

\[
0 = \dot{Q} - \dot{W} + \dot{m} \left( h_i - h_e + \frac{\vec{v}_i^2 - \vec{v}_e^2}{2000 \text{ J/kg}} \right)
\]

\[
0 = C_P (T_i - T_e) + \frac{\vec{v}_i^2 - \vec{v}_e^2}{2000 \text{ J/kg}}
\]

\[
T_e = T_i + \frac{\vec{v}_i^2 - \vec{v}_e^2}{C_P (2000 \text{ J/kg})}
\]

\[
T_e = 30^\circ C + \frac{(200 \text{ m/s})^2 - (20)^2}{1.004 \text{ kJ/kgK} (2000 \text{ J/kg})}
\]

\[
T_e = 49.7^\circ C = 322.9 \text{ K}
\]

Apply Second Law,

\[
0 = \sum \frac{\dot{S}}{T} + \dot{m} (s_i - s_e) + \dot{S}_{gen}
\]

\[
s_e = s_i \quad \text{Isentropic}
\]

\[
P_e = P_i \left( \frac{T_e}{T_i} \right)^{k/(k-1)}
\]

\[
P_e = 120 \text{ kPa} \left( \frac{322.9 \text{ K}}{303.15 \text{ K}} \right)^{1.4/0.4} = 149.7 \text{ kPa}
\]
One technique of operating a steam turbine at part load power output is to throttle the steam to a lower pressure before it enters the turbine. The steamline conditions are 2 MPa, 400°C, and the turbine exhaust pressure is fixed at 10 kPa. Assume the expansion inside the turbine to be reversible and adiabatic.

a. Determine the full-load specific work output of the turbine.

b. Find the pressure the steam must be throttled to for 80% of full-load output.

c. Show both processes in a T-s diagram.

**Given:** Line pressure and temperature. Exit Pressure. Turbine is adiabatic and reversible.

**Find:** Full-load specific work of turbine. Turbine inlet pressure for output power that is 80% of full-load power.

**Assumptions:** Steady flow. Valve is adiabatic. No changes in kinetic or potential energy.

**Solution:**

**Part A:**

State 'a' is the state entering the valve. State 'b' is the state leaving the valve and entering the turbine. State 'c' is the state leaving the turbine. Apply First Law to **Turbine**

\[
0 = \dot{Q} - \dot{W} + \dot{m} (h_b - h_c)
\]

\[
\frac{\dot{W}}{\dot{m}} = w = h_b - h_c
\]

Apply Second Law to **Turbine**,

\[
0 = \sum \frac{\dot{Q}}{T} - +\dot{m} (s_b - s_c) + \text{Gen}
\]

\[
s_b = s_c = 7.127 \text{ kJ/kgK}
\]

Define inlet and outlet state of **Turbine**.

<table>
<thead>
<tr>
<th>State b</th>
<th>State c</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_b = 2000$ kPa</td>
<td>$P_c = 10$ kPa</td>
</tr>
<tr>
<td>$T_b = 400^\circ$C</td>
<td>$s_b = s_c = 7.127 \text{ kJ/kgK}$</td>
</tr>
<tr>
<td>From CATT3</td>
<td>$h_b = 3248 \text{ kJ/kg}$</td>
</tr>
<tr>
<td>$h_c = 2258 \text{ kJ/kg}$</td>
<td></td>
</tr>
</tbody>
</table>

\[
w = h_b - h_c
\]

\[
w = 3248 \text{ kJ/kg} - 2258 \text{ kJ/kg}
\]

\[
w = 990 \text{ kJ/kg}
\]

**Part B:**

Apply First Law to **Valve**

\[
0 = \dot{Q} - \dot{W} + \dot{m} (h_a - h_b)
\]

\[
h_a = h_b
\]
It doesn’t matter what the valve position is (as long as it is not closed), the outlet enthalpy will always be equal to the inlet enthalpy.

Apply First Law to Turbine

\[ 0 = \dot{Q} - W + \dot{m} (h_b - h_c) \]

\[ h_c = h_b - w \]

\[ h_c = h_b - 80\% \left( 990 \text{ kJ/kg} \right) \]

\[ h_c = h_b - 792 \text{ kJ/kg} \]

Apply Second Law to Turbine,

\[ 0 = \sum \frac{\dot{Q}}{T} + \dot{m} (s_b - s_c) + \dot{S}_{\text{gen}} \]

\[ s_b = s_c \]

Define inlet and outlet state of Valve and Turbine.

<table>
<thead>
<tr>
<th>State a</th>
<th>State b</th>
<th>State c</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_a = 2000 \text{ kPa} )</td>
<td>( h_b = h_a = 3248 \text{ kJ/kg} )</td>
<td>( P_c = 10 \text{ kPa} )</td>
</tr>
</tbody>
</table>
| \( T_a = 400^\circ \text{C} \) | \( s_b = s_c \) | \( h_c = h_b - 792 \text{ kJ/kg} \) = \( 2456 \text{ kJ/kg} \)

From CATT3

\( s_a = 7.127 \text{ kJ/kgK} \)

\( h_a = 3248 \text{ kJ/kg} \)

\( s_c = 7.747 \text{ kJ/kgK} \)

Find \( P_b \) from \( s_b = s_c = 7.747 \text{ kJ/kgK} \) and \( h_b = h_a = 3248 \text{ kJ/kg} \)

\[ P_b \approx 510 \text{ kPa} \]