A piston/cylinder contains 0.1 kg of nitrogen at 100 kPa and 27°C that goes through a polytropic compression process with $n=1.25$ to a pressure of 250 kPa. How much work has the air done in the process? What is the heat transfer for the process?

**Given:** $P_1$, $T_1$, $m$, $n$, $P_2$

**Find:** $W$ and $Q$

**Assumptions:** Nitrogen can be treated as an ideal-gas. System is closed.

**Solution:**

Define states:

State 1

- $P_1 = 100$ kPa
- $T_1 = 400$ K
- $v_1 = \frac{RT_1}{P_1}$
- $v_1 = \frac{0.2968 kJ \cdot kg}{100 kPa} \cdot 300 K$
- $v_1 = 0.890 m^3/kg$

State 2

- $P_2 = 300$ kPa
- $v_2 = \left( \frac{P_1 v_1^n}{P_2} \right)^{\frac{1}{n}}$
- $v_2 = \left( \frac{100 kPa \left(0.890 m^3/kg\right)^{1.25}}{250 kPa} \right)^{\frac{1}{1.25}} = 0.428 m^3/kg$

Find Work:

$$W_{12} = \int_1^2 PdV$$

$$= \frac{P_2 V_2 - P_1 V_1}{1 - n} \Leftarrow \text{Polytropic process}$$

$$= m \frac{P_2 v_2 - P_1 v_1}{1 - n} \Leftarrow \text{Closed system}$$

$$= 0.1 kg \frac{(250 kPa)0.428 m^3/kg - (100 kPa)0.89 m^3/kg}{1 - 1.25}$$

$$W_{12} = -7.16 kJ$$

Use First Law to Find Heat:

$$E_2 - E_1 = Q_{12} - W_{12} + E_{in} - E_{exit}$$

$$m \left(u_2 - u_1\right) = Q_{12} - W_{12} \Leftarrow \text{Closed System}$$

$$Q_{12} = m \left(u_2 - u_1\right) + W_{12}$$

Need to evaluate $u_2 - u_1$. There are two ways to do this: 1) use table A7.1 to look up $u_1$ and $u_2$ or assume constant specific heat and use $u_2 - u_1 \approx C_v (T_2 - T_1)$. Either way we need to find $T_2$,

$$T_2 = \frac{P_2 v_2}{R} = \frac{(250 kPa) \left(0.428 m^3/kg\right)}{0.2968 kJ/kgK} = 360.3 K$$
Using table A7.1

\[ u_1 = 214.4 \frac{\text{kJ}}{\text{kg}} \]
\[ u_2 = 257.8 \frac{\text{kJ}}{\text{kg}} \] \Rightarrow \text{Interpolate on A7.1}

\[ Q_{12} = m(u_2 - u_1) + W_{12} \]
\[ Q_{12} = 0.1 \text{ kg} \left( 257.8 \frac{\text{kJ}}{\text{kg}} - 214.4 \frac{\text{kJ}}{\text{kg}} \right) - 7.165 \text{ kJ} \]
\[ Q_{12} = -2.82 \text{ kJ} \]

Assuming constant specific heat.

\[ u_2 - u_1 \approx C_v (T_2 - T_1) \]
\[ \approx 0.745 \frac{\text{kJ}}{\text{kgK}} (360.3 \text{ K} - 300 \text{ K}) \]
\[ \approx 45.0 \frac{\text{kJ}}{\text{kg}} \]

\[ Q_{12} = m(u_2 - u_1) + W_{12} \]
\[ Q_{12} = 0.1 \text{ kg} \left( 45.0 \frac{\text{kJ}}{\text{kg}} \right) - 7.165 \text{ kJ} \]
\[ Q_{12} = -2.69 \text{ kJ} \]
4.64

A piston/cylinder arrangement shown in Fig. P4.64 initially contains air at 150 kPa, 400°C. The setup is allowed to cool to the ambient temperature of 20°C.

a. Is the piston resting on the stops in the final state? What is the final pressure in the cylinder?

b. What is the specific work done by the air during this process?

Solution:

State 1: \( P_1 = 150 \text{ kPa}, \quad T_1 = 400°C = 673.2 \text{ K} \)

State 2: \( T_2 = T_0 = 20°C = 293.2 \text{ K} \)

For all states air behave as an ideal gas.

a) If piston at stops at 2, \( V_2 = V_1/2 \) and pressure less than \( P_{\text{lift}} = P_1 \)

\[
\Rightarrow P_2 = P_1 \times \frac{V_1}{V_2} \times \frac{T_2}{T_1} = 150 \text{ kPa} \times 2 \times \frac{293.2}{673.2} = 130.7 \text{ kPa} < P_1
\]

\[
\Rightarrow \text{Piston is resting on stops at state 2.}
\]

b) Work done while piston is moving at constant \( P_{\text{ext}} = P_1 \).

\[
W_2 = \int P_{\text{ext}} \, dV = P_1 (V_2 - V_1) \quad ; \quad V_2 = \frac{1}{2} V_1 = \frac{1}{2} m \frac{RT_1}{P_1}
\]

\[
W_2 = \frac{1}{2} \frac{m}{2} = \frac{RT_1}{P_1} (\frac{1}{2} - 1) = \frac{1}{2} \times 0.287 \text{ kJ/kg-K} \times 673.2 \text{ K}
\]

\[
= -96.6 \text{ kJ/kg}
\]

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A piston/cylinder contains 1 kg of liquid water at 20°C and 300 kPa. Initially the piston floats, similar to the setup in Problem 4.67, with a maximum enclosed volume of 0.002 m³ if the piston touches the stops. Now heat is added so a final pressure of 600 kPa is reached. Find the final volume and the work in the process.

Solution:

Take CV as the water which is a control mass: \( m_2 = m_1 = m \);

Table B.1.1: 20°C \( \Rightarrow P_{\text{sat}} = 2.34 \text{ kPa} \)

State 1: Compressed liquid \( v = v_f(20) = 0.001002 \text{ m}^3/\text{kg} \)

State 1a: \( v_{\text{stop}} = 0.002 \text{ m}^3/\text{kg} \), 300 kPa

State 2: Since \( P_2 = 600 \text{ kPa} > P_{\text{lift}} \) then piston is pressed against the stops

\[ v_2 = v_{\text{stop}} = 0.002 \text{ m}^3/\text{kg} \] and \( V = 0.002 \text{ m}^3 \)

For the given \( P : v_f < v < v_g \) so 2-phase \( T = T_{\text{sat}} = 158.85 \text{ °C} \)

Work is done while piston moves at \( P_{\text{lift}} = \text{constant} = 300 \text{ kPa} \) so we get

\[ W_2 = \int P \, dV = m \, P_{\text{lift}} \, (v_2 - v_1) = 1 \text{ kg} \times 300 \text{ kPa} \times (0.002 - 0.001002) \text{ m}^3 \]

\[ = 0.30 \text{ kJ} \]

\[ Q_2 = m(u_2 - u_1) + \Delta W_2 \]

\[ u_1 = u_f |_{T=20^\circ C} = 83.9 \frac{\text{kJ}}{\text{kg}} \]

\[ u_2 = 675.3 \frac{\text{kJ}}{\text{kg}} \]

\[ Q_2 = 1 \text{ kg}(675.3 \frac{\text{kJ}}{\text{kg}} - 83.9 \frac{\text{kJ}}{\text{kg}}) + 0.3 \text{ kJ} = 591.7 \text{ kJ} \]

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5.41

A rigid tank holds 0.75 kg ammonia at 70°C as saturated vapor. The tank is now cooled to 20°C by heat transfer to the ambient. Which two properties determine the final state. Determine the amount of work and heat transfer during the process.

C.V. The ammonia, this is a control mass.

Process: Rigid tank \( V = C \implies v = \text{constant} \) & \( W_2 = \int_1^2 \text{PdV} = 0 \)

Energy Eq.: \( U_2 - U_1 = \dot{Q}_2 - \dot{Q}_1 = \dot{Q}_2 \)

State 1: \( v_1 = 0.03787 \text{ m}^3/\text{kg}, \)
\( u_1 = 1338.9 \text{ kJ/kg} \)
State 2: \( T, v \implies \) two-phase (straight down in P-v diagram from state 1)

\[
x_2 = (v - v_f)/v_{fg} = (0.03787 - 0.001638)/0.14758 = 0.2455
\]
\[
u_2 = u_f + x_2 u_{fg} = 272.89 + 0.2455 \times 1059.3 = 532.95 \text{ kJ/kg}
\]

\( \dot{Q}_2 = m(u_2 - u_1) = 0.75 \text{ kg} (532.95 - 1338.9) \text{ kJ/kg} = -604.5 \text{ kJ} \)
A cylinder having a piston restrained by a linear spring (of spring constant 15 kN/m) contains 0.5 kg of saturated vapor water at 120°C, as shown in Fig. P5.55. Heat is transferred to the water, causing the piston to rise. If the piston cross-sectional area is 0.05 m², and the pressure varies linearly with volume until a final pressure of 500 kPa is reached. Find the final temperature in the cylinder and the heat transfer for the process.

Solution:
C.V. Water in cylinder.

Continuity: \( m_2 = m_1 = m \)

Energy Eq.5.11: \( m(u_2 - u_1) = 1Q_2 - 1W_2 \)

State 1: \((T, x)\) Table B.1.1 \( \Rightarrow \) \( v_1 = 0.89186 \) m³/kg, \( u_1 = 2529.2 \) kJ/kg

Process: \( P_2 = P_1 + \frac{ksm}{A_p^2} (v_2 - v_1) = 198.5 + \frac{15 \times 0.5}{(0.05)^2} (v_2 - 0.89186) \)

State 2: \( P_2 = 500 \) kPa \( \) and on the process curve (see above equation).

\( \Rightarrow \) \( v_2 = 0.89186 + (500 - 198.5) \times (0.05^2/7.5) = 0.9924 \) m³/kg

\((P, v)\) Table B.1.3 \( \Rightarrow \) \( T_2 = 803°C \); \( u_2 = 3668 \) kJ/kg

The process equation allows us to evaluate the work

\[ 1W_2 = \int PdV = \left( \frac{P_1 + P_2}{2} \right) m(v_2 - v_1) \]

\[ = \left( \frac{198.5 + 500}{2} \right) \text{kPa} \times 0.5 \text{ kg} \times (0.9924 - 0.89186) \text{ m}^3/\text{kg} = 17.56 \text{ kJ} \]

Substitute the work into the energy equation and solve for the heat transfer

\[ 1Q_2 = m(u_2 - u_1) + 1W_2 = 0.5 \text{ kg} \times (3668 - 2529.2) \text{ kJ/kg} + 17.56 \text{ kJ} = 587 \text{ kJ} \]
5.61

A rigid tank is divided into two rooms by a membrane, both containing water, shown in Fig. P5.61. Room A is at 200 kPa, \( v = 0.5 \) m\(^3\)/kg, \( V_A = 1 \) m\(^3\), and room B contains 3.5 kg at 0.5 MPa, 400°C. The membrane now ruptures and heat transfer takes place so the water comes to a uniform state at 100°C. Find the heat transfer during the process.

Solution:

C.V.: Both rooms A and B in tank.

Continuity Eq.: \[ m_2 = m_{A1} + m_{B1} \]

Energy Eq.: \[ m_2 u_2 - m_{A1} u_{A1} - m_{B1} u_{B1} = Q_2 - W_2 \]

State 1A: (P, v) Table B.1.2, \[ m_{A1} = \frac{V_A}{V_{A1}} = 1/0.5 = 2 \ kg \]

\[ x_{A1} = \frac{v - v_f}{v_{fg}} = \frac{0.5 - 0.001061}{0.88467} = 0.564 \]

\[ u_{A1} = u_f + x_{A1} u_{fg} = 504.47 + 0.564 \times 2025.02 = 1646.6 \text{ kJ/kg} \]

State 1B: Table B.1.3, \[ v_{B1} = 0.6173, \ u_{B1} = 2963.2, \ V_B = m_{B1} v_{B1} = 2.16 \text{ m}^3 \]

Process constant total volume: \[ V_{tot} = V_A + V_B = 3.16 \text{ m}^3 \] and \[ W_2 = 0 \]

\[ m_2 = m_{A1} + m_{B1} = 5.5 \text{ kg} \quad \Rightarrow \quad \frac{v_2}{m_2} = \frac{V_{tot}}{m_2} = 0.5746 \text{ m}^3/\text{kg} \]

State 2: \( T_2, v_2 \Rightarrow \) Table B.1.1 two-phase \( v_2 < v_g \)

\[ x_2 = \frac{v_2 - v_f}{v_{fg}} = \frac{0.5746 - 0.001044}{1.67185} = 0.343, \]

\[ u_2 = u_f + x_{fg} u_{fg} = 418.91 + 0.343 \times 2087.58 = 1134.95 \text{ kJ/kg} \]

Heat transfer is from the energy equation

\[ Q_2 = m_2 u_2 - m_{A1} u_{A1} - m_{B1} u_{B1} \]

\[ = (5.5 \times 1134.95 - 2 \times 1646.6 - 3.5 \times 2963.2) \text{ kg} \times \text{kJ/kg} \]

\[ = -7421 \text{ kJ} \]
5.122

Oxygen at 300 kPa, 100°C is in a piston/cylinder arrangement with a volume of 0.1 m³. It is now compressed in a polytropic process with exponent, \( n = 1.2 \), to a final temperature of 200°C. Calculate the heat transfer for the process.

Solution:

Continuity: \( m_2 = m_1 \)

Energy Eq.5.11: \( m(u_2 - u_1) = 1Q_2 - 1W_2 \)

State 1: \( T_1, P_1 \) & ideal gas, small change in \( T \), so use Table A.5

\[
\Rightarrow m = \frac{P_1V_1}{RT_1} = \frac{300 \times 0.1 \text{ m}^3}{0.25983 \times 373.15} = 0.309 \text{ kg}
\]

Process: \( PV^n = \text{constant} \)

\[
1W_2 = \frac{1}{1-n} (P_2V_2 - P_1V_1) = \frac{mR}{1-n} (T_2 - T_1) = \frac{0.309 \times 0.25983}{1 - 1.2} (200 - 100)
\]

\[
= -40.2 \text{ kJ}
\]

\[
1Q_2 = m(u_2 - u_1) + 1W_2 = mC_v(T_2 - T_1) + 1W_2
\]

\[
= 0.3094 \times 0.662 (200 - 100) - 40.2 = -19.7 \text{ kJ}
\]

If you didn’t notice that you can use \( mR(T_2 - T_1) = P_2V_2 - P_1V_1 \),
then you could also find \( P_2 \) and \( V_2 \)

\[
P_2V_2 = RT_2, \quad P_2V_2^n = P_1V_1^n
\]

Two equations, two unknowns.
4.106

A large condenser (heat exchanger) in a power plant must transfer a total of 100 MW from steam running in a pipe to sea water being pumped through the heat exchanger. Assume the wall separating the steam and seawater is 4 mm of steel, conductivity 15 W/m K and that a maximum of 5°C difference between the two fluids is allowed in the design. Find the required minimum area for the heat transfer neglecting any convective heat transfer in the flows.

Solution:

Steady conduction through the 4 mm steel wall.

\[ \dot{Q} = k \ A \ \frac{\Delta T}{\Delta x} \Rightarrow A = \frac{\dot{Q} \ \Delta x}{k \Delta T} \]

\[ A = 100 \times 10^6 \text{ W} \times 0.004 \text{ m} \left(15 \text{ W/mK} \times 5 \text{ K}\right) \]

\[ = 480 \text{ m}^2 \]
4.107

A 2 m² window has a surface temperature of 15°C and the outside wind is blowing air at 2°C across it with a convection heat transfer coefficient of $h = 125$ W/m²K. What is the total heat transfer loss?

Solution:

$$\dot{Q} = h \ A \ \Delta T = 125 \ \text{W/m}^2\text{K} \times 2 \ \text{m}^2 \times (15 - 2) \ \text{K} = 3250 \ \text{W}$$

as a rate of heat transfer out.
A radiant heat lamp is a rod, 0.5 m long and 0.5 cm in diameter, through which 400 W of electric energy is deposited. Assume the surface has an emissivity of 0.9 and neglect incoming radiation. What will the rod surface temperature be?

Solution:

For constant surface temperature outgoing power equals electric power.

\[
\dot{Q}_{\text{rad}} = \varepsilon \sigma A T^4 = \dot{Q}_{\text{el}}
\]

\[
T^4 = \frac{\dot{Q}_{\text{el}}}{\varepsilon \sigma A} = \frac{400 \text{ W}}{(0.9 \times 5.67 \times 10^{-8} \text{ W/} \text{m}^2 \text{K}^4 \times 0.5 \times \pi \times 0.005 \text{ m}^2)}
\]

\[
= 9.9803 \times 10^{11} \text{ K}^4 \Rightarrow T = 1000 \text{ K OR } 725 \text{ °C}
\]

\[
\dot{Q}(c_{q0}) = \varepsilon \sigma A (T_s - T_a)
\]

\[
T_s = \frac{\dot{Q}_{\text{rad}}}{\varepsilon \sigma A} + T_a
\]

\[
T_s = \frac{400 \text{ W}}{0.9 (5.67 \times 10^{-8} \text{ W/m}^2 \text{K}^4) 0.5 \pi \times 0.005 \text{ m}} + 293 \text{ K}
\]

\[
= 1005 \times 10^4 \text{ K}
\]

\[
T_s = 1005 \Delta 1 \text{ K}
\]