Estimate the transfer function for each of the four systems.

The magnitude ratio of system $G_1$ (solid green) “breaks” down at 20 dB/dec at $\omega = 1$ rad/s and again at $\omega = 200$ rad/s so the transfer function is:

$$G_1 = \kappa \frac{1}{(s+1)(s+200)} = \kappa \frac{1}{s^2 + 201s + 200}$$

To find $\kappa$ we use the steady state gain. From the figure, the $M_r$ at low frequency is $\approx -6$ dB. So the steady state gain is,

$$K_{ss} \approx 10^{-6/20} = 0.50$$

For the steady state gain of the transfer function to be 0.50, $\kappa$ needs to be 100. The approximate transfer function is:

$$G_1 = \frac{100}{s^2 + 201s + 200}$$

The magnitude ratio of the system $G_2$ (red dash-dot) “breaks” up at at 20 dB/dec at $\omega = 1$ rad/s and then “breaks” down at 20 dB/dec at $\omega = 10$ rad/s so the transfer function is:

$$G_2 = \kappa \frac{s + 1}{(s+10)}$$

To find $\kappa$ we use the steady state gain. From the figure, the $M_r$ at low frequency is $\approx 0$ dB. So the steady state gain is,

$$K_{ss} \approx 10^0 = 1$$
For the steady state gain of the transfer function to be 1, $\kappa$ needs to be 10. The approximate transfer function is:

$$G_2 = \frac{10(s + 1)}{s + 10} = \frac{10s + 10}{s + 10}$$

The magnitude ratio of system $G_3$ (blue dot) “breaks” down at at 40 dB/dec at $\omega \approx 3$ (actually at $\omega = \sqrt{10}$). The “bump” on the magnitude plot as well as the relatively quick transition of -180° of the phase indicate a pair of complex poles. The transfer function has the form:

$$G_3 = \frac{K_{ss}\omega_n}{s^2 + 2\zeta\omega_ns + \omega_n^2}$$

$\omega_n = \sqrt{10}$. $\zeta$ can be approximated by the size of the “bump” in the $M_R$ figure. The “bump” is about 10 dB high which corresponds to a damping ratio of about 0.15 (it was really $\frac{1}{2\sqrt{10}} \approx 0.158$). With these values the transfer function becomes:

$$G_3 = \frac{K_{ss}10}{s^2 + 2(0.158)\sqrt{10}s + 10} = \frac{K_{ss}10}{s^2 + 1s + 10}$$

From the figure, the $M_r$ at low frequency is $\approx 20$ dB. So the steady state gain is,

$$K_{ss} \approx 10^{20/20} = 10$$

The approximate transfer function is:

$$G_3 = \frac{100}{s^2 + s + 10}$$

The magnitude ratio of system $G_4$ (black dashed) has a slope of -20 dB/dec for frequencies below 10 rad/s and a slope of -40 dB/dec for frequencies above 10 rad/s so the transfer function is:

$$G_4 = \frac{\kappa}{(s + 0)(s + 10)} = \frac{\kappa}{s(s + 10)}$$

There is no steady state gain so $\kappa$ must be found some other way. To find $\kappa$ we just evaluate $|G(j\omega)|$ at any frequency. Any frequency will work, but we will get more accurate results if we stay away from “break” frequencies... in this case we don’t want to use a frequency too close to 10 rad/s. Choosing $\omega = 1000$ rad/s we can read the $M_R$ from the figure. $M_R \approx -100$ dB. This means $|G(1000j)| \approx 10^{-100/20} = 10^{-5}$. Using the transfer function to evaluate the $|G(1000j)|$:

$$|G(1000j)| = \frac{\kappa}{|1000j||1000j + 10|} \approx \frac{\kappa}{(1000)(1000)} = \kappa(10^{-6})$$

so,

$$\kappa(10^{-6}) = 10^{-5}$$

and,

$$\kappa = 10$$

so,

$$G_4 = \frac{10}{s(s + 10)} = \frac{10}{s^2 + 10s}$$
The Code I used to create the figure.

clear

%The transfer functions
G1=tf(1,[1,1])*tf(100,[1 200])
G2=tf([10 10],[1 10])
G3=tf(100,[1 1 10])
G4=tf(10,[1 10 0])

%Making the figure
figure(1);clf
set(1,'Position',[100,100,1200,800]);
h=bodeplot(G1,'g',G2,'r-.',G3,':',G4,'k--');
legend({'G_1','G_2','G_3','G_4'})
grid on
ylims = getoptions(h,'YLim');
ylims{1}=[-100,40];
ylims{2}=[-180,90];
setoptions(h,'YLimMode','manual','YLim',ylims);
set(findall(gcf,'type','line'),'linewidth',3);

G1 =

100
----------------
 s^2 + 201 s + 200

Continuous-time transfer function.

G2 =

10 s + 10
---------
 s + 10

Continuous-time transfer function.

G3 =

100
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 s^2 + s + 10

Continuous-time transfer function.

G4 =

10
\[ s^2 + 10 \, s \]

Continuous-time transfer function.