A room is heated with a 1500 W electric heater. How much power can be saved if a heat pump with a COP of 2.0 is used instead?

Assume the heat pump has to deliver 1500 W as the $\dot{Q}_H$.

Heat pump: $\beta' = \dot{Q}_H/\dot{W}_{IN}$

$$\dot{W}_{IN} = \dot{Q}_H/\beta' = \frac{1500}{2} = 750 \text{ W}$$

So the heat pump requires an input of 750 W thus saving the difference

$$\dot{W}_{saved} = 1500 \text{ W} - 750 \text{ W} = 750 \text{ W}$$
7.22

An air-conditioner discards 5.1 kW to the ambient with a power input of 1.5 kW. Find the rate of cooling and the coefficient of performance.

Solution:

In this case \( \dot{Q}_H = 5.1 \text{ kW} \) goes to the ambient so

Energy Eq.: \( \dot{Q}_L = \dot{Q}_H - \dot{W} = 5.1 - 1.5 = 3.6 \text{ kW} \)

\[
\beta_{REFRIG} = \frac{\dot{Q}_L}{\dot{W}} = \frac{3.6}{1.5} = 2.4
\]
Consider a heat engine and heat pump connected as shown in figure P7.42. Assume \( T_{H1} = T_{H2} > T_{amb} \) and determine for each of the three cases if the setup satisfy the first law and/or violates the 2\(^{nd}\) law.

<table>
<thead>
<tr>
<th>( \dot{Q}_m )</th>
<th>( \dot{Q}_{L1} )</th>
<th>( \dot{W}_1 )</th>
<th>( \dot{Q}_{M2} )</th>
<th>( \dot{Q}_{L2} )</th>
<th>( \dot{W}_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>a 6</td>
<td>4</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>b 6</td>
<td>4</td>
<td>2</td>
<td>5</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>c 3</td>
<td>2</td>
<td>1</td>
<td>4</td>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>

\[
\frac{\dot{W}_{\text{net}}}{\dot{W}_1 - \dot{W}_2} = \begin{cases} 1, \\ 0 \end{cases}
\]

**First Law**
\[
0 = \dot{Q}_{H1} + \dot{Q}_{L2} - \dot{Q}_{L1} - \dot{Q}_{Rc} - \dot{W}_{\text{net}}
\]

a) \[
6 + 2 - 4 - 3 - 1 = 0 \quad \checkmark
\]
b) \[
6 + 4 - 4 - 5 - 1 = 0 \quad \checkmark
\]
c) \[
3 + 3 - 2 - 4 - 0 = 0 \quad \checkmark
\]

**Second Law**
\[
\sum \frac{\dot{Q}}{T} \leq 0
\]

\[
\frac{\dot{Q}_{H1}}{T_H} + \frac{\dot{Q}_{L2}}{T_H} - \frac{\dot{Q}_{L1}}{T_A} - \frac{\dot{Q}_{Rc}}{T_A} \leq 0
\]

a) \[
\frac{6}{T_H} + \frac{2}{T_H} - \frac{4}{T_H} - \frac{3}{T_M} = \frac{6 - 4}{T_H} + \frac{2 - 3}{T_A} = \frac{2}{T_H} - \frac{1}{T_A} \leq 0 \quad \text{(possible)}
\]
b) \[ \frac{6}{T_H} + \frac{4}{T_A} - \frac{4}{T_A} - \frac{5}{T_H} = \frac{6-5}{T_H} + \frac{0}{T_L} \leq 0 \]

\[ \frac{5}{T_H} \leq 0 \leftarrow \text{not true, impossible} \]

Net heat transfer to low temp is zero.
So system converts all of the heat from the high temp into work. This is not possible.

\[ \frac{3}{T_H} + \frac{3}{T_A} - \frac{2}{T_A} - \frac{4}{T_H} = \frac{3-2}{T_H} + \frac{3-2}{T_A} = \frac{-1}{T_H} + \frac{1}{T_A} \leq 0 \]

Heat transfer in the wrong direction. not true since \( T_A < T_H \)

7.62

We propose to heat a house in the winter with a heat pump. The house is to be maintained at 20°C at all times. When the ambient temperature outside drops to -10°C, the rate at which heat is lost from the house is estimated to be 25 kW. What is the minimum electrical power required to drive the heat pump?

Solution:

Minimum power if we assume a Carnot cycle

Energy equation for the house (steady state):

\[ \dot{Q}_H = \dot{Q}_{\text{leak}} = 25 \text{ kW} \]

\[ \beta' = \frac{\dot{Q}_H}{\dot{Q}_{\text{leak}}} = \frac{T_H}{T_H - T_L} = \frac{293.2}{20 - (-10)} = 9.773 \Rightarrow \dot{W}_{\text{IN}} = \frac{25}{9.773} = 2.56 \text{ kW} \]
An ideal gas Carnot cycle with air in a piston cylinder has a high temperature of 1200 K and a heat rejection at 400 K. During the heat addition the volume triples. Find the two specific heat transfers \( q \) in the cycle and the overall cycle efficiency.

Solution:

The \( P-v \) diagram of the cycle is shown to the right.

From the integration along the process curves done in the main text we have Eq. 7.7

\[
q_H = R \frac{T_H}{v_1} \ln \left( \frac{v_2}{v_1} \right) \\
= 0.287 \times 1200 \ln(3) \\
= 378.4 \text{ kJ/kg}
\]

Since it is a Carnot cycle the knowledge of the temperatures gives the cycle efficiency as

\[
\eta_{TH} = 1 - \frac{T_L}{T_H} = 1 - \frac{400}{1200} = 0.667
\]

from which we can get the other heat transfer from Eq. 7.4

\[
q_L = q_H \frac{T_L}{T_H} = 378.4 \times \frac{400}{1200} = 126.1 \text{ kJ/kg}
\]
A combination of a heat engine driving a heat pump (see Fig. P7.106) takes waste energy at 50°C as a source $\dot{Q}_{w1}$ to the heat engine rejecting heat at 30°C. The remainder $\dot{Q}_{w2}$ goes into the heat pump that delivers a $\dot{Q}_H$ at 150°C. If the total waste energy is 5 MW find the rate of energy delivered at the high temperature.

Solution:

Waste supply: $\dot{Q}_{w1} + \dot{Q}_{w2} = 5$ MW

Heat Engine:

$\dot{W} = \eta \dot{Q}_{w1} = (1 - \frac{T_{L1}}{T_{H1}}) \dot{Q}_{w1}$

Heat pump:

$\dot{W} = \dot{Q}_H / \beta_{HP} = \dot{Q}_{w2} / \beta'$

$= \dot{Q}_{w2} / [T_{H1} / (T_H - T_{H1})]$}

Equate the two work terms:

$(1 - \frac{T_{L1}}{T_{H1}}) \dot{Q}_{w1} = \dot{Q}_{w2} \times (T_H - T_{H1}) / T_{H1}$

Substitute $\dot{Q}_{w1} = 5$ MW - $\dot{Q}_{w2}$

$(1 - 303.15 / 323.15)(5 - \dot{Q}_{w2}) = \dot{Q}_{w2} \times (150 - 50) / 323.15$

$20 (5 - \dot{Q}_{w2}) = \dot{Q}_{w2} \times 100 \quad \Rightarrow \quad \dot{Q}_{w2} = 0.8333$ MW

$\dot{Q}_{w1} = 5 - 0.8333 = 4.1667$ MW

$\dot{W} = \eta \dot{Q}_{w1} = 0.06189 \times 4.1667 = 0.258$ MW

$\dot{Q}_H = \dot{Q}_{w2} + \dot{W} = 1.09$ MW

(For the heat pump $\beta' = 423.15 / 100 = 4.23$)
7.106 METHOD II

\[ \sum \frac{\dot{Q}}{T} = 0 \]

\[ \dot{Q}_{w1} + \dot{Q}_{w2} = 5 \text{ MW} = \dot{Q}_L + \dot{Q}_H \]

\[ \dot{Q}_L = 5 \text{ MW} - \dot{Q}_H \]

\[ 0 = \sum \frac{\dot{Q}}{T} = \frac{\dot{Q}_{w1} + \dot{Q}_{w2}}{323K} - \frac{\dot{Q}_L}{303K} - \frac{\dot{Q}_H}{423K} \]

\[ = \frac{5 \text{ MW}}{323K} - \frac{5 \text{ MW}}{303K} + \frac{\dot{Q}_A}{303K} - \frac{\dot{Q}_A}{423K} \]

\[ 5 \text{ MW} \left( \frac{1}{303K} - \frac{1}{323K} \right) = \dot{Q}_H \left( \frac{1}{303K} - \frac{1}{423K} \right) \]

\[ \dot{Q}_H = 5 \text{ MW} \frac{\left( \frac{1}{303K} - \frac{1}{323K} \right)}{\left( \frac{1}{303K} - \frac{1}{423K} \right)} = 1.09 \text{ MW} \]
8.46 CO₂  \[ R = 0.188 \text{ mJ} \text{ mK}^{-1} \text{ kg}^{-1} \text{ k} = 1.289 \]
\[ C_v = 0.653 \frac{\text{kJ}}{\text{kg} \text{K}} \]
\[ m = 1 \text{ kg} \]

**State 1**

\[ T_1 = 120^\circ \text{C} = 393 \text{K} \]
\[ P_1 = 1400 \text{ kPa} \]

\[ \Delta \text{ke} = \Delta \text{pe} = 0 \]

\[ m(u_2 - u_1) = \overrightarrow{u}_2 - \overrightarrow{u}_1 \]

\[ W'_2 = -m(u_2 - u_1) = -m C_v (T_2 - T_1) \]
\[ = -1 \text{kg} (0.653 \frac{\text{kJ}}{\text{kg} \text{K}}) (347 \text{K} - 393 \text{K}) \]
\[ W'_2 = +30 \text{ kJ} \]

**State 2**

\[ P_2 = 800 \text{ kPa} \]
\[ T_2 = T_1 \left( \frac{P_2}{P_1} \right)^{k/v_k} \]
\[ T_2 = 393 \text{K} \left( \frac{800 \text{ kPa}}{1400 \text{ kPa}} \right)^{0.289/1.289} \approx 347 \text{K} \]