A steam turbine receives steam from two boilers. One flow is 5 kg/s at 3 MPa, 700°C and the other flow is at 15 kg/s at 800 kPa, 500°C. The exit state is 10 kPa with a quality of 96%. Find the total power out of the adiabatic turbine.

**Given:** Inlet and outlet conditions of steam turbine.

**Find:** Power output of turbine.

**Assumptions:** Steady flow process, negligible changes in kinetic and potential energy, \( Q = 0 \) (adiabatic).

**Solution:**

Conservation of mass:

\[
\frac{dm}{dt} = \sum \dot{m}_i - \sum \dot{m}_e \\
0 = \dot{m}_1 + \dot{m}_2 - \dot{m}_3 \\
\dot{m}_3 = \dot{m}_1 + \dot{m}_2
\]

Conservation of Energy:

\[
\frac{dE}{dt} = \dot{Q} - \dot{W} + \sum \dot{m}_i (h_i + ke_i + pe_i) - \sum \dot{m}_e (h_e + ke_e + pe_e) \\
0 = 0 - \dot{W} + \dot{m}_1 h_1 + \dot{m}_2 h_2 - \dot{m}_3 h_3 \\
\dot{W} = \dot{m}_1 h_1 + \dot{m}_2 h_2 - \dot{m}_3 h_3
\]

Define States:

<table>
<thead>
<tr>
<th>State 1</th>
<th>State 2</th>
<th>State 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_1 = 3 \text{ MPa} )</td>
<td>( P_2 = 800 \text{ kPa} )</td>
<td>( P_3 = 10 \text{ kPa} )</td>
</tr>
<tr>
<td>( T_1 = 700^\circ\text{C} )</td>
<td>( T_2 = 500^\circ\text{C} )</td>
<td>( x_3 = 0.96 )</td>
</tr>
<tr>
<td>( h_1 = 3912 \text{ kJ/kg} )</td>
<td>( h_2 = 3481 \text{ kJ/kg} )</td>
<td>( h_3 = 191.8\text{kJ/kg+(0.96)2392.8kJ/kg= 2489 kJ/kg} )</td>
</tr>
<tr>
<td>( \dot{m}_1 = 5 \text{ kg/s} )</td>
<td>( \dot{m}_2 = 15 \text{ kg/s} )</td>
<td>( \dot{m}_3 = \dot{m}_1 + \dot{m}_2 = 20 \text{ kg/s} )</td>
</tr>
</tbody>
</table>

\[
\dot{W} = 5 \text{ kg/s}(3912 \text{ kJ/kg}) + 15 \text{ kg/s}(3481 \text{ kJ/kg}) - 20 \text{ kg/s}(2489 \text{ kJ/kg}) = 22.0 \text{ MW}
\]
A compressor receives 0.05 kg/s R-410a at 200 kPa, -20°C and 0.1 kg/s R-410a at 400 kPa, 0°C. The exit flow is at 1000 kPa, 60°C. Assume adiabatic, neglect kinetic energies, and find the required, power input.

**Given:** Inlet and outlet conditions of a compressor.

**Find:** Required power input to compressor.

**Assumptions:** Steady flow process, negligible changes in kinetic and potential energy, \( \dot{Q} = 0 \) (adiabatic).

**Solution:**

Conservation of mass:

\[
\left( \frac{dm}{dt} \right)_{CV} = \sum \dot{m}_i - \sum \dot{m}_e \\
0 = \dot{m}_1 + \dot{m}_2 - \dot{m}_3 \\
\dot{m}_3 = \dot{m}_1 + \dot{m}_2
\]

Conservation of Energy:

\[
\left( \frac{dE}{dt} \right) = \dot{Q} - \dot{W} + \sum \dot{m}_i (h_i + ke_i + pe_i) - \sum \dot{m}_e (h_e + ke_e + pe_e) \\
0 = 0 - \dot{W} + \dot{m}_1 h_1 + \dot{m}_2 h_2 - \dot{m}_3 h_3 \\
\dot{W} = \dot{m}_1 h_1 + \dot{m}_2 h_2 - \dot{m}_3 h_3
\]

Define States:

<table>
<thead>
<tr>
<th>State 1</th>
<th>State 2</th>
<th>State 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_1 ) = 200 kPa</td>
<td>( P_2 ) = 400 kPa</td>
<td>( P_3 ) = 1000 kPa</td>
</tr>
<tr>
<td>( T_1 ) = -20°C</td>
<td>( T_2 ) = 0°C</td>
<td>( T_3 ) = 60°C</td>
</tr>
<tr>
<td>( h_1 ) = 278.7 kJ/kg</td>
<td>( h_2 ) = 290.4 kJ/kg</td>
<td>( h_3 ) = 335.7 kJ/kg</td>
</tr>
<tr>
<td>( \dot{m}_1 ) = 0.05 kg/s</td>
<td>( \dot{m}_2 ) = 0.1 kg/s</td>
<td>( \dot{m}_3 ) = ( \dot{m}_1 ) + ( \dot{m}_2 ) = 0.15 kg/s</td>
</tr>
</tbody>
</table>

\[
\dot{W} = 0.5 \text{ kg/s}(278.7 \text{ kJ/kg}) + 0.1 \text{ kg/s}(290.4 \text{ kJ/kg}) - 0.15 \text{ kg/s}(335.7 \text{ kJ/kg}) = -7.38 \text{ kW}
\]
6.85 A heat exchanger is used to cool an air flow from 800 K to 360 K, with both states at 1 MPa. The coolant is a water flow at 15°C at 0.1 MPa. If the water leaves as saturated vapor, find the ratio of the flow rates \( \dot{m}_{\text{water}} / \dot{m}_{\text{air}} \).

**Given:** Inlet and outlet conditions of a heat exchanger.

**Find:** The ratio of flow rates needed to satisfy the inlet/outlet conditions.

**Assumptions:** Steady flow process, streams do not mix, air can be treated as an ideal gas, negligible changes in kinetic and potential energy, \( Q = 0 \) (to surroundings).

**Solution:**

Conservation of Energy:

\[
\left( \frac{dE}{dt} \right) = \dot{Q} - \dot{W} + \sum \dot{m}_i (h_i + ke_i + pe_i) - \sum \dot{m}_e (h_e + ke_e + pe_e) \\
0 = 0 + \dot{m}_{\text{air}} (h_1 - h_2) + \dot{m}_{\text{water}} (h_3 - h_4)
\]

\[
\frac{\dot{m}_{\text{water}}}{\dot{m}_{\text{air}}} = \frac{h_1 - h_2}{h_3 - h_3}
\]

Define States:

<table>
<thead>
<tr>
<th>State 1</th>
<th>State 2</th>
<th>State 3</th>
<th>State 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_1 = 1.00 \text{ MPa} )</td>
<td>( P_2 = 1.00 \text{ MPa} )</td>
<td>( P_3 = 100 \text{ kPa} )</td>
<td>( P_4 = 100 \text{ kPa} )</td>
</tr>
<tr>
<td>( T_1 = 800 \text{ K} )</td>
<td>( T_2 = 360 \text{ K} )</td>
<td>( T_3 = 15^\circ \text{C} )</td>
<td>( x_4 = 1 )</td>
</tr>
<tr>
<td>( h_1 = 822.2 \text{ kJ/kg} )</td>
<td>( h_2 = 360.9 \text{ kJ/kg} )</td>
<td>( P &gt; P_{\text{sat}}(T=15^\circ \text{C}) ) : subcooled</td>
<td>( h_4 = 2675.5 \text{ kJ/kg} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( h_3 \approx u_f(T=T_3) + P v_f(T=T_3) )</td>
<td>( h_4 \approx 62.98 \text{ kJ/kg} + (100\text{kPa})0.001001\text{m}^3/\text{kg} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( h_3 \approx 63.08 \text{ kJ/kg} )</td>
<td>( h_4 \approx 2675.5 - 63.08 = 0.177 )</td>
</tr>
</tbody>
</table>

\[
\frac{\dot{m}_{\text{water}}}{\dot{m}_{\text{air}}} = \frac{h_1 - h_2}{h_3 - h_3} = \frac{822.2 - 360.9}{2675.5 - 63.08} = 0.177
\]
Two air flows are combined to a single flow. One flow is $1 \text{ m}^3/\text{s}$ at $20^\circ \text{C}$ and the other $2 \text{ m}^3/\text{s}$ at $200^\circ \text{C}$, both at $100 \text{ kPa}$. They mix without any heat transfer to produce an exit flow at $100 \text{ kPa}$. Neglect kinetic energies and find the exit temperature and volume flow rate.

**Given:** Inlet temperature, pressure, and volumetric flow rates. Outlet pressure

**Find:** Outlet temperature and volumetric flow rates.

**Assumptions:** Steady flow process, air can be treated as an ideal gas with constant specific heat, negligible changes in kinetic and potential energy, $\dot{Q} = 0$ (to surroundings).

**Solution:**

Conservation of Mass:

$$\left( \frac{dm}{dt} \right)_{\text{CV}} = \sum \dot{m}_i - \sum \dot{m}_e$$

$$0 = \dot{m}_1 + \dot{m}_2 - \dot{m}_3$$

$$\dot{m}_3 = \dot{m}_1 + \dot{m}_2$$

Conservation of Energy:

$$\left( \frac{dE}{dt} \right) = \dot{Q} - \dot{W} + \sum \dot{m}_i (h_i + ke_i + pe_i) - \sum \dot{m}_e (h_e + ke_e + pe_e)$$

$$0 = 0 - \dot{m}_1 h_1 + \dot{m}_2 h_2 - \dot{m}_3 h_3$$

$$0 = 0 - 0 + \dot{m}_1 h_1 + \dot{m}_2 h_2 - (\dot{m}_1 + \dot{m}_2) h_3$$

$$0 = \dot{m}_1 (h_1 - h_3) + \dot{m}_2 (h_2 - h_3)$$

$$0 = \dot{m}_1 C_p (T_1 - T_3) + \dot{m}_2 C_p (T_2 - T_3)$$

$$T_3 = \frac{\dot{m}_1 C_p T_1 + \dot{m}_2 C_p T_2}{\dot{m}_1 C_p + \dot{m}_2 C_p}$$

$$T_3 = \frac{\dot{m}_1 T_1 + \dot{m}_2 T_2}{\dot{m}_1 + \dot{m}_2}$$

Define States:
<table>
<thead>
<tr>
<th>State 1</th>
<th>State 2</th>
<th>State 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_1 = 100,\text{kPa}$</td>
<td>$P_2 = 100,\text{kPa}$</td>
<td>$P_3 = 100,\text{kPa}$</td>
</tr>
<tr>
<td>$T_1 = 20^\circ\text{C}$</td>
<td>$T_1 = 200^\circ\text{C}$</td>
<td></td>
</tr>
<tr>
<td>$V_1 = 1,\text{m}^3/\text{s}$</td>
<td>$V_2 = 2,\text{m}^3/\text{s}$</td>
<td></td>
</tr>
<tr>
<td>$v_1 = RT_1/P_1$</td>
<td>$v_2 = RT_2/P_2$</td>
<td></td>
</tr>
<tr>
<td>$v_1 = (0.287\text{kJ/kgK})(293\text{K})/100,\text{kPa}$</td>
<td>$v_2 = (0.287\text{kJ/kgK})(473\text{K})/100,\text{kPa}$</td>
<td></td>
</tr>
<tr>
<td>$v_1 = 0.841,\text{m}^3/\text{kg}$</td>
<td>$v_2 = 1.36,\text{m}^3/\text{kg}$</td>
<td></td>
</tr>
<tr>
<td>$\dot{m}_1 = \dot{V}_1/v_1$</td>
<td>$\dot{m}_2 = \dot{V}_2/v_2$</td>
<td></td>
</tr>
<tr>
<td>$\dot{m}_1 = 1.19,\text{kg/s}$</td>
<td>$\dot{m}_2 = 1.47,\text{kg/s}$</td>
<td></td>
</tr>
</tbody>
</table>

\[
\dot{m}_3 = \dot{m}_1 + \dot{m}_2
\]

\[
\dot{m}_3 = 1.19\,\text{kg/s} + 1.47\,\text{kg/s}
\]

\[
\dot{m}_3 = 2.66\,\text{kg/s}
\]

\[
T_3 = \frac{\dot{m}_1 T_1 + \dot{m}_2 T_2}{\dot{m}_1 + \dot{m}_2}
\]

\[
T_3 = \frac{(1.19\,\text{kg/s})(20^\circ\text{C}) + (1.47\,\text{kg/s})(200^\circ\text{C})}{2.66\,\text{kg/s}}
\]

\[
T_3 = 119^\circ\text{C}
\]

\[
\dot{V}_3 = \dot{m}_3 v_3
\]

\[
\dot{V}_3 = \dot{m}_3 RT_3/P_3
\]

\[
\dot{V}_3 = 2.66\,\text{kg/s} \left( \frac{(0.287\text{kJ/kgK})(392\text{K})}{100\,\text{kPa}} \right)
\]

\[
\dot{V}_3 = 2.66\,\text{kg/s} \left( \frac{1.125\,\text{m}^3/\text{kg}}{100\,\text{kPa}} \right)
\]

\[
\dot{V}_3 = \frac{2.99\,\text{m}^3}{\text{s}}
\]

Note that $\dot{V}_3 = \dot{V}_1 + \dot{V}_2$ is just a coincidence in this case and not always true.
An open feedwater heater in a power plant heats 4 kg/s water at 45°C, 100 kPa by mixing it with steam from the turbine at 100 kPa, 250°C. Assume the exit flow is saturated liquid at 100 kPa and find the mass flow rate from the turbine.

**Given:** Inlet and exit conditions. One inlet mass flow rate. **Find:** The other inlet mass flow rate.

**Assumptions:** Steady flow process, negligible changes in kinetic and potential energy, $\dot{Q} = 0$ (to surroundings).

**Solution:**

**Conservation of mass:**

$$\frac{dm}{dt}_{CV} = \sum \dot{m}_i - \sum \dot{m}_e$$

$$0 = \dot{m}_1 + \dot{m}_2 - \dot{m}_3$$

$$\dot{m}_3 = \dot{m}_1 + \dot{m}_2$$

**Conservation of Energy:**

$$\left(\frac{dE}{dt}\right) = \dot{Q} - \dot{W} + \sum \dot{m}_i(h_i + ke_i + pe_i) - \sum \dot{m}_e(h_e + ke_e + pe_e)$$

$$0 = 0 - 0 + \dot{m}_1 h_1 + \dot{m}_2 h_2 - \dot{m}_3 h_3$$

$$0 = \dot{m}_1 h_1 + \dot{m}_2 h_2 - (\dot{m}_1 + \dot{m}_2) h_3$$

$$0 = \dot{m}_1 (h_1 - h_3) + \dot{m}_2 (h_2 - h_3)$$

$$\dot{m}_2 = \dot{m}_1 \frac{h_3 - h_1}{h_2 - h_3}$$

Define States:

<table>
<thead>
<tr>
<th>State 1</th>
<th>State 2</th>
<th>State 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_1 = 100$ kPa</td>
<td>$P_2 = 100$ kPa</td>
<td>$P_3 = 100$ kPa</td>
</tr>
<tr>
<td>$T_1 = 45°C$</td>
<td>$T_2 = 250°C$</td>
<td>$x_3 = 0$</td>
</tr>
<tr>
<td>$h_1 \approx u_f</td>
<td>45°C + P v_f</td>
<td>45°C$</td>
</tr>
<tr>
<td>$h_1 \approx 188.4$kJ/kg + (100kPa)(0.001010m³)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$h_1 \approx 188.5$kJ/kg</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
\[ \dot{m}_2 = \frac{\dot{m}_1 (h_3 - h_1)}{h_2 - h_3} \]

\[ \dot{m}_2 = 4\text{kg/s} \frac{417.4\text{kJ/kg} - 188.5\text{kJ/kg}}{2974\text{kJ/kg} - 417.4\text{kJ/kg}} \]

\[ \dot{m}_2 = 0.358\text{kg/s} \]
7.7 A combination of two refrigerator cycles is shown in the figure. Find the overall COP as a function of COP\(_1\) and COP\(_2\).

Given: COPs of individual refrigerators. 
Find: COP of the combined refrigerator. 
Assumptions: Steady flow process. 
Solution:

\[
\beta_{\text{Total}} = \frac{\dot{Q}_L}{\dot{W}_1 + \dot{W}_2} \quad (1)
\]

\[
\beta_1 = \frac{\dot{Q}_L}{\dot{W}_1} \Rightarrow \dot{W}_1 = \frac{\dot{Q}_L}{\beta_1} \quad (2)
\]

\[
\beta_2 = \frac{\dot{Q}_M}{\dot{W}_2} \Rightarrow \dot{W}_2 = \frac{\dot{Q}_M}{\beta_2} \quad (3)
\]

Plugging equations (2) and (3) into equation (1):

\[
\beta_{\text{Total}} = \frac{\dot{Q}_L}{\beta_1 + \beta_2 - \beta_1 \beta_2} \quad (4)
\]

\[
\dot{Q}_M = \dot{W}_1 + \dot{Q}_L = \frac{\dot{Q}_L}{\beta_1} + \dot{Q}_L \quad (5)
\]

Plugging equations (5) into equation (4) and divide through by \(\dot{Q}_L\):

\[
\beta_{\text{Total}} = \frac{\dot{Q}_L}{\frac{\dot{Q}_L}{\beta_1} + \dot{Q}_L} = \frac{1}{\frac{1}{\beta_1} + \frac{1}{\beta_2}}
\]

Multiply by \(\frac{\beta_1 \beta_2}{\beta_1 \beta_2}\):

\[
\beta_{\text{Total}} = \frac{\beta_1 \beta_2}{1 + \beta_1 + \beta_2}
\]
7.16 Calculate the efficiency of the steam power plant given in Example 6.9.

\[
\eta = \frac{\dot{W}_{\text{net}}}{\dot{Q}_{\text{in}}} = \frac{\dot{W}_T - \dot{W}_P}{\dot{Q}_b} = \frac{w_T - w_P}{Q_b} = \frac{640.7 \text{kJ/kg} - 4 \text{kJ/kg}}{2831 \text{kJ/kg}} = 0.225 = 22.5\%
\]

7.32 For each of the cases below, determine if the heat engine satisfies the first law and if it violates the second law.

a. \( \dot{Q}_H = 6 \text{kW} \quad \dot{Q}_L = 4 \text{kW} \quad \dot{W} = 2 \text{kW} \)

b. \( \dot{Q}_H = 6 \text{kW} \quad \dot{Q}_L = 0 \text{kW} \quad \dot{W} = 6 \text{kW} \)

c. \( \dot{Q}_H = 6 \text{kW} \quad \dot{Q}_L = 2 \text{kW} \quad \dot{W} = 5 \text{kW} \)

d. \( \dot{Q}_H = 6 \text{kW} \quad \dot{Q}_L = 6 \text{kW} \quad \dot{W} = 0 \text{kW} \)

Solution

a. \( \dot{Q}_H = \dot{Q}_L + \dot{W} \), Satisfies first law. Does not violate second law.

b. \( \dot{Q}_H = \dot{Q}_L + \dot{W} \), Satisfies first law. Violates second law because all of the heat is converted to work.

c. \( \dot{Q}_H \neq \dot{Q}_L + \dot{W} \), Violates first law. Does not violate second law.

d. \( \dot{Q}_H = \dot{Q}_L + \dot{W} \), Satisfies first law. Does not violate second law.

7.44 Calculate the thermal efficiency of a Carnot-cycle heat engine operating between reservoirs at 300°C and 45°C. Compare the results to that of Problem 7.16.

Given: Temperatures of two thermal reservoirs.

Find: Maximum efficiency of a heat engine operating between the two temperatures.

Solution

\[
\eta_{\text{Max}} = 1 - \frac{T_L}{T_H} = 1 - \frac{318K}{573} = 0.445 = 44.5\%
\]

The efficiency in 7.16 is lower than this because there are loses and most of the heat is added at temperatures below 300°C, and the heat is rejected at temperatures above 45°C.